Since $n_n$ typically $10^{15} - 10^{16} \text{ cm}^{-3}$ currently, need $B > \sim 200 \text{ G}$, so electrons may already be magnetized in some expts.

**Ratios: (gyrofrequency/collision freq), (gyroradius/ device L)**

<table>
<thead>
<tr>
<th>Fraction</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>electrons</td>
<td>$\frac{\omega_{ce}}{v_e} \sim \frac{6 \times 10^{14} B(\text{G})}{n_n(\text{cm}^{-3})} \gg 1$</td>
</tr>
<tr>
<td></td>
<td>$\rho_e \sim \frac{3.4}{B(\text{G})} \text{ cm} \ll L$</td>
</tr>
<tr>
<td>ions</td>
<td>$\frac{\omega_{ci}}{v_i} \sim 10^{12} \frac{B(\text{G})}{n_n(\text{cm}^{-3})} \gg 1$</td>
</tr>
<tr>
<td></td>
<td>$\rho_i \sim \frac{140}{B(\text{G})} \text{ cm} \ll L$</td>
</tr>
</tbody>
</table>

Since $m_i >> m_e$, $T_i << T_e$ (while $\sigma_{in} \sim 10 \sigma_{en}$), need larger $B$ to magnetize the ions:

$$B > 0.5 - 2 \text{ T with current } n_n \text{ (argon)}$$
**dust**

\( \rho_d \sim 2\text{g/cc} \),

\( \text{Ar}, T_n \sim 300\text{ K} \)

\[
B \gg \frac{R(\mu m)n_m\left(\text{cm}^{-3}\right)}{2 \times 10^8 |\phi(V)|}
\]

\[
B \gg \frac{3 \times 10^5 \sqrt{T_d(eV)R(\mu m)}}{|\phi(V)|L(cm)}
\]

**Larger B needed as** \( R \uparrow, \ n_m \uparrow, \ T_d \uparrow, \ \phi \downarrow, \ L \downarrow \)**

\[ \text{e.g., } \phi \sim -4\text{ V}, \ R \sim 0.1\ \mu\text{m}, \ P \sim 10\ \text{Pa}, \quad B \gg 30\ \text{T} \]

\[ \text{as above, with } P \sim 0.13\ \text{Pa}, \quad B \gg 0.4\ \text{T} \]
Use of small dust having surface plasmon resonance?

- **Background**: Surface Plasmon ($n_e$ wave) at interface

Polarizability of small sphere

$$
\alpha \sim R^3 \left( \frac{\varepsilon - \varepsilon_0}{\varepsilon + 2\varepsilon_0} \right)
$$

Re $\varepsilon = -2 \varepsilon_0$ \hspace{1cm} (Im $\varepsilon << 1$)

**Surface plasmon** - optical, near-UV (Au, Ag)
Possible way to see nano-sized dust in plasma

- Absorption and scattering enhanced at SP frequency

\[ Q_{\text{abs}} \sim 4x \text{Im} \left( \frac{\varepsilon - 1}{\varepsilon + 2} \right), \quad Q_{\text{scat}} \sim \frac{8}{3} x^4 \left| \frac{\varepsilon - 1}{\varepsilon + 2} \right|^2, \quad x = \frac{2\pi R}{\lambda} \ll 1 \]

- Use of dust with SP
  - ‘see’ nanoparticle transport in plasma
  - ‘see’ waves, etc. in plasma with magnetized dust (\( R \sim 20 \) nm, B \( \sim 2T \))
Possible diagnostic for dust waves in magnetized dusty plasma

- **Nm sized dust could be magnetized**
  
  e.g. Ar, $T_e \sim 2$ eV, $B \sim 2$ T, $P < 3$ Pa

  \[
  \rho_d < \sim 1 \text{ cm}, \quad \omega_{cd} > \nu_d, \quad F_{\text{Lorentz}} > F_{\text{gravity}}
  \]

- ‘See’ waves by enhanced absorption by dust with SP

  Compression - dark; rarefaction - light

  \[
  n_d \sim 10^9 \text{ cm}^{-3}, \quad \lambda \sim 350 \text{ nm (Re } \varepsilon \sim -2) \]

  Absorption coeff. $\sim 0.1$ /cm

Rosenberg & Shukla, *APL* 86, 2005
Theory has been done for dust wave instabilities under the following conditions, but not aware of experiments to test these:

**Unmagnetized dust, magnetized ions**

- Hall current \((E \times B)\) driven instabilities
- Drift instabilities

**Magnetized dust**

- Electrostatic dust cyclotron instability (higher harmonics too)
- Waves in plasma crystals
Possible constraint related to Electrostatic Dust Cyclotron waves

EDC wave: in long $\lambda$ limit, $k_\perp \rho_d << 1$,

$$\omega \sim \omega_{cd} (1+\Delta), \quad \Delta \sim k_\perp^2 c_{sd}^2 / 2 \omega_{cd}^2 << 1,$$

where the dust acoustic speed $c_{sd} \sim \lambda_{Di} \omega_{pd}$ (for $T_e >> T_i$)

$$k_\perp \left( cm^{-1} \right) < 0.01 \quad B(G) \left( \frac{T_i (eV) n_d m_d}{n_i m_p} \right)^{-1/2}$$

e.g. $T_i \sim 0.2 \text{ eV}$, $n_d/n_i \sim 10^{-4}$, $m_d/m_p \sim 2 \times 10^9$ ($R \sim 0.1 \mu m$),

$B \sim 2 \text{ T} \quad \Rightarrow \quad \lambda_\perp > 7 \text{ cm} \ (\lambda_\parallel > \lambda_\perp), \quad \text{larger device ?}$
**Estimates of B for controlling motion of paramagnetic dust**

Induced magnetic moment

\[ M = R^3 \left( \frac{\mu - 1}{\mu + 2} \right) B \]

Ratio of forces: attractive magnetic dipole-dipole to repulsive Coulomb

\[
\frac{F_M}{F_e} \sim \frac{6M^2}{d^4} \frac{d^2}{Z_d e^2} \sim 0.005 \left( \frac{R^2 (\mu m) B (G)}{d (\mu m) \phi(V)} \right)^2 \left( \frac{\mu - 1}{\mu + 2} \right)^2 \propto \frac{R^4 B^2}{d^2}
\]

Tune grain interactions via \( F_M \), using superparamagnetic grains

e.g., \( \phi \sim -4 \text{ V} \), \( R \sim 5 \text{ \( \mu \)m} \), \( d \sim 100 \text{ \( \mu \)m} \), \( \mu \sim 3 \), \( B \sim 200 \text{ G} \), \( F_M/F_e \sim 0.1 \)

as above with \( R \sim 0.3 \text{ \( \mu \)m} \), \( F_M/F_e \sim 0.1 \) requires \( B \sim 5 \text{ T} \)

as \( R \downarrow \), \( B \uparrow \)
Magnetic packing force: magnetic grains move to regions of max. $B$

**Ratio of forces: magnetic packing to attractive magnetic dipole-dipole**

\[
\frac{F_{MP}}{F_M} \sim \frac{\nabla(MB)}{6M^2} \sim \frac{\left(\frac{d}{R}\right)^3 \left(\frac{\mu + 2}{\mu - 1}\right) d\nabla B}{6B}
\]

e.g., as in previous example, $R \sim 5 \, \mu m$, $d \sim 100 \, \mu m$, $\mu \sim 3$, $B \sim 200 \, G$, $\frac{F_{MP}}{F_M} > 1$ if $\frac{\partial B}{\partial z} \sim 6 \, G/cm$

**Levitate [1]:** $F_{MP}$, grav. force both proportional to $R^3$

\[
B\nabla B \sim \left(\frac{\mu + 2}{\mu - 1}\right) \frac{4\pi \rho d \, g}{3}
\]

e.g., $\mu \sim 3$, $B \sim 1000 \, G$, $\frac{\partial B}{\partial z} \sim 20 \, G/cm$, $\rho_d \sim 2g/cc$

Some physics issues that could be studied at large $B \sim T$

**Charging**: Debye sphere, anisotropic, $E \times B$ of electrons ($\rho_e \sim R$)?

**Forces**: effect of magnetized ions ($\rho_i \sim 200 \mu m$) on ion drag?

**Fusion**: how does $B$ affect structure of ablation cloud around large grain?

**Waves**: observe EDC waves and instabilities?

**Crystal**: how do magnetized ions & electrons affect structure and waves? (e.g., $\rho_i \sim$ lattice spacing, $E \times B$ motion in sheath)

**Crystal**: can crystal form if dust is magnetized (smaller $R$, lower $Z_d$)?
Some physics issues that could be studied at B < T

(magnetized ions and/or electrons, unmagnetized dust)

Waves: observe dust wave instabilities driven by electron or ion cross-field drifts, including Hall current instabilities, and drift wave instabilities?

Paramag. dust: tune behavior of dust acoustic waves (attractive force along B, repulsive perpendicular to B)?

Paramag. dust: tune spacings and structure in a plasma crystal by changing magnitude and orientation of B?

Paramag. dust: use $F_M + F_e$ force to levitate one type of grain, $F_e$ to levitate other type, form binary crystal?